## BRIEF COMMUNICATIONS

## Structure Refinement of $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$

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Received July 30, 1987; in revised form January 4, 1988


#### Abstract

The crystal structure of $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$ at 294 K was investigated by X -ray diffraction and refined to an $R$ value of 0.025 for 2735 reflections ( 1492 independent reflections). It belongs to the space group $P 6_{3} / m m c$ with $a=5.5587(1), c=21.8929(3) \AA, V=585.83(1) \AA^{3}$, and $Z=2, D_{x}=3.79 \mathrm{~g} / \mathrm{cm}^{3}$, and $\mu(\mathrm{MoK} \alpha)=15.86 \mathrm{~cm}^{-1}$. Three different models are refined for the bipyramidal $\mathrm{Al}^{3+}$ ions: (1) the central atom model, (2) the central atom model with anharmonic vibration, and (3) the split atom model. The results indicate that the split atom model where the $\mathrm{Al}^{3+}$ ion occupies statistically one of two equivalent sites is the real structure at room temperature. © 1988 Academic Press. Inc.


## Introduction

The crystal structure of $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$ was studied by Kato and Saalfeld (I), and confirmed to be isostructural with the mineral magnetoplumbite. The structure belongs to the hexagonal space group $P 6_{3} / \mathrm{mmc}$. In this structure Al atoms are distributed over three crystallographically independent octahedral sites, one tetrahedral site, and one trigonal bipyramidal site. However, an ambiguity remains about the exact location of the Al atom in the trigonal bipyramidal site. Namely, there are at least two probable models; in one model, a five-coordinated Al atom locates at the $2 b$ site on the mirror plane of the trigonal bipyramid (central atom model) and, in the other, two half-
atoms locate at two equivalent 4 e sites (split atom model) displaced from the mirror plane.

This report describes a result of structure refinement on $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$ carried out to elucidate the precise structural features of the bipyramidal cation.

## Experimental and Results

A single crystal was grown by the traveling solvent floating zone method using two halogen lamps as the heat source. The solvent with composition of $\mathrm{CaO} \cdot 4 \mathrm{Al}_{2} \mathrm{O}_{3}$ was used between a sintered rod and a seed crystal. A sintered rod with composition of $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$ was used as the seed crystal. The growth rate was $1.5 \mathrm{~mm} / \mathrm{hr}$.

A hexagonal platy crystal with thickness of 0.12 mm and hexagonal edge length of 0.13 mm was picked out from the crushed pieces of the grown crystal and used for intensity measurements. It was transparent and colorless. The space group $P 6_{3} / m m c$ was confirmed with Weissenberg photographs. Intensities were measured on a four-circle diffractometer (Rigaku AFC5UD) with MoK radition ( $\lambda=0.71069 \AA$ ) monochromated with pyrolytic graphite at 294 K . The $\omega$-scan mode was employed with a scanning speed of $2 \% / \mathrm{min}$. Scan widths were determined according to the formula $(1.4+0.5 \tan \theta)^{\circ}$. The intensities were corrected for Lorentz-polarization factors and for absorption effects, taking the crystal shape into consideration.

The 2625 reflections which satisfied the condition $\left|F_{\mathrm{o}}\right| \geqq 3 \sigma\left(\left|F_{\mathrm{o}}\right|\right)$ were obtained within the range $2 \theta \leqq 120^{\circ}, 0 \leqq h \leqq 13,0 \leqq$ $k \leqq 13$, and $-53 \leqq l \leqq 0$ ( 3520 reflections were measured, of which 731 were unobserved; there were 1492 unique observed reflections). In addition, 110 strong reflections were measured in the full reciprocal space to correct anisotropic extinction effects. A total of 2735 reflections were used for the subsequent refinements. The cell parameters were determined from 144 reflections in the range $71.2 \leqq 2 \theta \leqq 85.8^{\circ}$ by the least-squares procedure. They are $a=$ 5.5587(1) and $c=21.8929(3) \AA$.

Atomic scattering factors for $\mathrm{Ca}^{2+}$ and $\mathrm{Al}^{3+}$ ions and dispersion correction factors were taken from "International Tables for X-ray Crystallography" (16). The scattering factor given by Tokonami (2) was used for $\mathrm{O}^{2-}$ ions. The structure was refined with a modified version of the full-matrix leastsquares program LINKT85, which has a function for extinction correction after Becker and Coppens (3) and also has a function for anharmonic vibration refinement by Tanaka and Marumo (4). The refinement was carried out on two models. The first refinement was carried out on the
central atom model, starting from the atomic parameters given for $\mathrm{BaO} \cdot 6 \mathrm{Fe}_{2} \mathrm{O}_{3}$ by Obradors et al. (5). The least-squares calculations assuming an anisotropic type I extinction effect gave a little smaller $R$ ( $=0.039$ ) and $R_{\mathrm{w}}(=0.036)$ values than those ( 0.042 and 0.038 ) assuming an anisotropic type II extinction effect after some cycles of refinement. Therefore, further refinement was continued on the assumption of the type I extinction effects and gave the final $R$ and $R_{\mathrm{w}}$ values of 0.028 and 0.027 , respectively. The second refinement was carried out on the split atom model, like Obradors et al. tried on $\mathrm{BaO} \cdot 6 \mathrm{Fe}_{2} \mathrm{O}_{3}$, assuming the anisotropic extinction effects of type I. The least-squares calculations for this model gave the final $R$ and $R_{\mathrm{w}}$ values of, respectively, 0.025 and 0.026 , which are little smaller than those for the central atom model. Final parameters of the split atom model are given in Table I.

## Discussions

The refinements with the bipyramidal $\mathrm{Al}(2)$ atom in the 2 b site (central atom model) gave a $U_{33}$ value of $0.0489 \AA^{2}$ for $\mathrm{Al}(2)$ atom, which is extremely larger than those for the other Al atoms. The difference Fourier map of this model showed a negative peak with the depth of $-1.96 \mathrm{e} / \AA^{3}$ at $\mathrm{Al}(2)$ site and two positive peaks with heights of $1.73 \mathrm{e} / \AA^{3}$ at $0.2 \AA$ apart from $\mathrm{Al}(2)$ site along the $c$ axis. On the other hand, the rcfinements with the bipyramidal $\mathrm{Al}(2)$ atom in the 4 e sites (split atom model) reduced the $U_{33}$ value for $\mathrm{Al}(2)$ atom to $0.0110 \AA^{2}$, though the value is still larger than those for the other Al atoms. The difference Fourier map gave only quite small residual densities in this case. The negative peak at $\mathrm{Al}(2)$ site reduced the depth to $-0.07 \mathrm{e} / \AA^{3}$ and two positive peaks with heights of $0.10 \mathrm{e} / \AA^{3}$ were observed at $0.4 \AA$ apart from $\mathrm{Al}(2)$ site along the $c$ axis.

The peaks on the difference Fourier map

TABLE I
Positional and Thermal ( $\times 10^{5}$ ) Parameters

| Atom | Position | $x$ |  | $y$ | $z$ | $U_{11}$ | $U_{n 7}$ | $U_{23}$ | $U_{n 3}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ca | 2 d | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $1273(12)$ | $U_{11}$ | $1353(20)$ | 0 |  |
| $\mathrm{Al}(1)$ | 2 a | 0 |  | 0 | 0 | $335(13)$ | $U_{11}$ | $256(20)$ | 0 |
| $\mathrm{Al}(2)$ | 4 e | 0 |  | 0 | $0.25771(7)$ | $265(16)$ | $U_{11}$ | $1098(71)$ | 0 |
| $\mathrm{Al}(3)$ | 4 f | $\frac{1}{3}$ | $\frac{2}{3}$ | $0.02808(3)$ | $276(9)$ | $U_{11}$ | $395(14)$ | 0 |  |
| $\mathrm{Al}(4)$ | $4 \mathbf{f}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $0.19102(3)$ | $327(9)$ | $U_{11}$ | $333(14)$ | 0 |  |
| $\mathrm{Al}(5)$ | 12 k | $0.16855(3)$ | $0.33710(7)$ | $-0.10914(1)$ | $306(6)$ | $262(8)$ | $435(8)$ | $23(6)$ |  |
| $\mathrm{O}(1)$ | 4 e | 0 | 0 | $0.14907(5)$ | $326(17)$ | $U_{11}$ | $509(30)$ | 0 |  |
| $\mathrm{O}(2)$ | 4 f | $\frac{2}{3}$ | $\frac{1}{3}$ | $0.05461(5)$ | $392(17)$ | $U_{11}$ | $319(26)$ | 0 |  |
| $\mathrm{O}(3)$ | 6 h | $0.18107(10)$ | $0.36214(19)$ | $\frac{1}{4}$ | $712(20)$ | $337(24)$ | $485(22)$ | 0 |  |
| $\mathrm{O}(4)$ | 12 k | $0.15505(7)$ | $0.31010(13)$ | $0.05213(3)$ | $395(11)$ | $303(15)$ | $394(14)$ | $-23(12)$ |  |
| $\mathrm{O}(5)$ | 12 k | $0.50325(7)$ | $1.00650(14)$ | $0.14912(3)$ | $358(11)$ | $420(17)$ | $509(15)$ | $138(13)$ |  |

Note. The thermal parameters are of the form: $\exp \left[-2 \pi^{2}\left(U_{11} h^{2} a^{* 2}+U_{22} k^{2} b^{* 2}+U_{32} l^{2} c^{* 2}+2 U_{12} h k a^{*} h^{*}+\right.\right.$ $\left.\left.2 U_{13}^{\prime} h l a^{*} c^{*}+2 U_{23} k l h^{*} c^{*}\right)\right] ; 2 U_{12}=U_{22} ; 2 U_{13}=U_{23}$.
for the central atom model may be ascribed to an anharmonic thermal vibration of $\mathrm{Al}(2)$ atom. Refinement was, therefore, carried out by introducing anharmonic potential parameters. The site symmetry of $\mathrm{Al}(2)$ atom is $\overline{6} m 2$ and the temperature factor including anharmonic terms up to the fourth order was given by Tanaka and Marumo (4). The number of parameters increased is 6 in comparison with the simple central atom model. The final $R$ and $R_{\mathrm{w}}$ values were 0.025 and 0.026 with this refinement, respectively. The difference Fourier map showed a negative peak with the depth of $-0.65 \mathrm{e} / \AA^{3}$ at $\mathrm{Al}(2)$ site and two positive peaks with heights of $0.24 \mathrm{e} / \AA^{3}$ at $0.2 \AA$ from $\mathrm{Al}(2)$ site along the $c$ axis. Although refinement on the central atom model allowing anharmonic thermal vibration for $\mathrm{Al}(2)$ atom gave the same $R$ value as the split atom model, the former evidently gave larger peaks on difference density maps than the latter. Thus the present study seems to support that the split atom model where $\mathrm{Al}(2)$ atom occupies statistically one of two cquivalent sites in the 4 e positions is the real structure of $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$ at room temperature.

Table II lists interatomic distances within
$\mathrm{AlO}_{5}$ trigonal bipyramids observed in magnetoplumbite-type (6-11) and andalu-site-type structures ( 12,13 ). The corresponding distances in $\mathrm{BaO} \cdot 6 \mathrm{Fe}_{2} \mathrm{O}_{3}$ are also given for comparison. Mössbauer measurements for $\mathrm{BaO} \cdot 6 \mathrm{Fe}_{2} \mathrm{O}_{3}$ (14) revealed that the $\mathrm{Fe}^{3+}$ ion at the bipyramidal site occupies one of two equivalent positions of the 4 e site at temperatures below 80 K and that it jumps between the two 4 e sites at higher temperatures than about 80 K . The $\mathrm{FeO}_{5}$ bipyramid in $\mathrm{BaO} \cdot 6 \mathrm{Fe}_{2} \mathrm{O}_{3}$ is slightly larger than the $\mathrm{AlO}_{5}$ bipyramid in the aluminates and the equatorial $\mathrm{O}-\mathrm{O}$ distances in $\mathrm{BaO} \cdot 6 \mathrm{Fe}_{2} \mathrm{O}_{3}$ are also longer than those in the aluminates by about $0.2 \AA$. This $\mathrm{O}-\mathrm{O}$ separation acts as a potential barrier when the split ion jumps between the two sites. In the case of $\mathrm{BaO} \cdot 6 \mathrm{Fe}_{2} \mathrm{O}_{3}$, the opening of the oxygen equilateral triangle has a size of $0.47 \AA$, if the $\mathrm{O}^{2-}$ ions are assumed to be rigid spheres with radii of $1.39 \AA$. Since $O(3)$ atom is shared by five cations, this value is expected for $O(3)$ ion as the effective ionic radius (15). Similarly the size of the opening is calculated to be $0.35 \AA$ for $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$. Since the ionic radius of $\mathrm{Al}^{3+}$ ion ( $0.48 \AA$ for fivefold coordination) is smaller than that for $\mathrm{Fe}^{3+}$ ion ( $0.58 \AA$ ) by

TABLE II
Comparison of the Atomic Distances (Å) in $\mathrm{AlO}_{5}$ Trigonal Bipyramid

| Sample | Average Al-O distance | Al to oxygen on the mirror plane distance | O--O distance of oxygen triangle on the mirror plane | $\begin{gathered} \text { Separation } \\ \text { of } \\ \mathrm{Al}-\mathrm{Al} \end{gathered}$ | Remarks | References |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.974 | 1.751 | 3.020 | 0.338 | Split atom model | Present study |
| $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.919 | 1.740 | 3.015 |  | Central atom model | (l) |
| $\mathrm{SrO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.949 | 1.755 | 3.040 |  | Central atom model | (6) |
| $\mathrm{La}_{2} \mathrm{O}_{3} \cdot 2 \mathrm{MgO} \cdot 11 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.938 | 1.762 | 3.051 |  | Central atom model | (7) |
| $\mathrm{I}_{2} \mathrm{O}_{3} \cdot 2 \mathrm{MnO} \cdot 11 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.945 | 1.763 | 3.028 | 0.445 | Split atom model | (8) |
| $\mathrm{La}_{2} \mathrm{O}_{3} \cdot 0.75 \mathrm{CoO} \cdot 11 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.966 | 1.809 | 3.100 | 0.519 | Split atom model | (8) |
| $\mathrm{La}_{2} \mathrm{O}_{3} \cdot 11 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.939 | 1.732 | 2.970 | 0.490 | Split atom model | (9) |
| $\mathrm{Nd}_{2} \mathrm{O}_{3} \cdot 11 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.943 | 1.755 | 3.014 | 0.457 | Split atom model | (10) |
| $\mathrm{Nd}_{2} \mathrm{O}_{3} \cdot \mathrm{Na}_{2} \mathrm{O} \cdot 23 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.927 | 1.748 | 3.013 | 0.383 | Split atom model | (11) |
| $\mathrm{BaO} \cdot 6 \mathrm{Fe}_{2} \mathrm{O}_{3}$ | $2.039^{\text {a }}$ | $1.867^{\circ}$ | 3.220 | $0.340{ }^{\text {a }}$ | Split atom model | (5) |
| $2 \mathrm{~B}_{2} \mathrm{O}_{3} \cdot 9 \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.88 | $1.80^{h}$ | $3.05^{\text {b }}$ |  | (andalusite type) | (12) |
| $\mathrm{SiO}_{2} \cdot \mathrm{Al}_{2} \mathrm{O}_{3}$ | 1.836 | $1.822^{b}$ | $3.137 \%$ |  | (andalusite type) | (13) |

${ }^{4} \mathrm{Fe}-\mathrm{O}$ or $\mathrm{Fe}-\mathrm{Fe}$ distance.
${ }^{\circ}$ Average value.
about $0.10 \AA$, the $\mathrm{Al}^{3+}$ ions in bipyramidal sites for $\mathrm{CaO} \cdot 6 \mathrm{Al}_{2} \mathrm{O}_{3}$ are probably not at rest in a statical disorder configuration but dynamically change their sites. However, in order to get further information about the dynamical character of $\mathrm{Al}(2)$ atom, it is necessary to analyze the structure at various temperatures including temperatures below 80 K .

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